Q) Find the nature of roots of the following quadratic equations. If the real roots exist, find them.

ii) $3x^2 - 4\sqrt{3}\,x + 4 = 0$

Sol: $3x^2 - 4\sqrt{3}\,x + 4 = 0$

On comparing with $ax^2 + bx + c = 0$,

we get; $a = 3$, $b = -4\sqrt{3}$, $c = 4$

$b^2 - 4ac = (-4\sqrt{3})^2 - 4\,(3)\,(4)$

$= 16 \times 3 - 48$

$= 48 - 48$

$= 0$

$b^2 - 4ac = 0$

$\therefore$ The two roots are real and equal

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-(-4\sqrt{3}) \pm 0}{2 \times 3}$

$\therefore\ x = \dfrac{4\sqrt{3} + 0}{6}$ or $x = \dfrac{4\sqrt{3} - 0}{6}$

$\therefore\ x = \dfrac{\cancel{4}^{2}\sqrt{3}}{\cancel{6}_{3}}$ or $x = \dfrac{\cancel{4}^{2}\sqrt{3}}{\cancel{6}_{3}}$

$\therefore\ x = \dfrac{2\sqrt{3}}{3}$ or $x = \dfrac{2\sqrt{3}}{3}$

$\therefore\ x = \dfrac{2}{\sqrt{3}}$ or $x = \dfrac{2}{\sqrt{3}}$

$\dfrac{2\sqrt{3}}{3} \Rightarrow \dfrac{2\cancel{\sqrt{3}}}{\sqrt{3} \times \cancel{\sqrt{3}}} = \dfrac{2}{\sqrt{3}}$

Find the nature of roots of the following quadratic equations. If the real roots exist, find them.

i) $2x^2 - 3x + 5 = 0$

Sol: $2x^2 - 3x + 5 = 0$

On comparing with $ax^2 + bx + c = 0$,

we get; $a = 2$, $b = -3$, $c = 5$

$b^2 - 4ac = (-3)^2 - 4\,(2)\,(5)$

$= 9 - 40$

$b^2 - 4ac = -31$

$b^2 - 4ac < 0$

Hence roots of the quadratic equation are not real

$2 \times 5 = 10$

10

$-5 \quad +2$

-7

$2x^2 - 3x + 5 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-(-3) \pm \sqrt{-31}}{2(2)} = \dfrac{3 \pm \sqrt{-31}}{4}$

$=$

Find the nature of roots of the following quadratic equations. If the real roots exist, find them.

ii) $2x^2 - 6x + 3 = 0$

$2x^2 - 6x + 3 = 0$

On comparing with $ax^2 + bx + c = 0$,

we get; $a = 2$, $b = -6$, $c = 3$

$b^2 - 4ac = (-6)^2 - 4\,(2)\,(3)$

$= 36 - 24$

$= 12$

$b^2 - 4ac > 0$

Hence roots of the quadratic equation are real and distinct.

$\therefore\ x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-(-6) \pm \sqrt{12}}{2(2)}$

$= \dfrac{6 \pm 2\sqrt{3}}{4}$

$= \dfrac{2\,(3 \pm \sqrt{3})}{4}$

$= \dfrac{3 \pm \sqrt{3}}{2}$

$\therefore\ x = \dfrac{3 + \sqrt{3}}{2}$ or $x = \dfrac{3 - \sqrt{3}}{2}$

$2 \,|\, 12$

$2 \,|\, 6$

$3 \,|\, 3$

$2\sqrt{3}$

$= \dfrac{6 \pm 2\sqrt{3}}{4} = \dfrac{\cancel{2}(3 \pm \sqrt{3})}{\cancel{4}_{2}}$

$\therefore$ The roots of the given quadratic equations are $\dfrac{3 + \sqrt{3}}{2}$ and $\dfrac{3 - \sqrt{3}}{2}$

Find the value of k :

If one root of the quadratic equation $3x^2 - 8x - (2k+1) = 0$ is 7 times the other, Find the value of k.

Let one root be α

other root $\beta = 7\alpha$

$a = 3$

$b = -8$

$c = 2k+1$

Sum of the roots $= \dfrac{-b}{a} = \dfrac{-(-8)}{3} = \dfrac{8}{3}$

$\alpha + 7\alpha = \dfrac{8}{3}$

$8\alpha = \dfrac{8}{3}$

$\alpha = \dfrac{\cancel{8}}{3} \times \dfrac{1}{\cancel{8}} = \dfrac{1}{3}$

$\boxed{\therefore\ \alpha = \dfrac{1}{3}}$

pdt of the roots $= \dfrac{c}{a}$

$\therefore\ \alpha(7\alpha) = \dfrac{-(2k+1)}{3}$

$\dfrac{1}{3} \times 7 \times \dfrac{1}{3} \Rightarrow$

$\Rightarrow \dfrac{7}{\cancel{9}_{3}} = \dfrac{-(2k+1)}{\cancel{3}}$

$\dfrac{7}{3} = -(2k+1)$

$7 = -(2k+1)\,3$

$7 = -(6k+3)$

$\alpha = \frac{8}{3}$

$\alpha = \frac{\cancel{8}}{3} \times \frac{1}{\cancel{8}} = \frac{1}{3}$ $\boxed{\therefore \alpha = \frac{1}{3}}$

$7 = -(2k+1)3$

$7 = -(6k+3)$

$7 = -6k-3$

$7+3 = -6k$

$\frac{\cancel{10}5}{-\cancel{6}3} = k$ $\boxed{\therefore k = \frac{-5}{3}}$

② Find the value of k for which the roots of the quadratic equation $5x^2-10x+k=0$ are real and equal.

$D = b^2-4ac \;\; = \; \geq$

Given :- $5x^2-10x+k=0$

Comparing = $a=5,\ b=-10,\ c=k.$

Since the roots are real and equal.

$D=0$

$\therefore D = b^2-4ac=0$

$\Rightarrow (-10)^2-4(5)(k) = 0$

$100-20k=0$

$-20k=-100$

$k = \frac{-100}{-20} = 5$

$\boxed{\therefore k=5}$

3) Find the values of k for which the equation $9x^2+3kx+4=0$ has real roots.

$D \geq 0$

D { = real root equal; > real root Distinct (unequal) }

real root unequal (Distinct) $D>0$

real root equal $D=0$

Ans

Given quad eqn $\Rightarrow 9x^2+3kx+4=0$

Comparing = $a=9,\ b=3k,\ c=4$

Now $D = b^2-4ac$

$D = (3k)^2-4(9)(4) = 9k^2-144$

Since, roots of the eqn are real

$D \geq 0$

$D \geq 0$

$9k^2-144 \geq 0$

$9(k^2-16) \geq 0 \quad (\because 9 \neq 0)$

$k^2-16 \geq 0$

$k^2-4^2 \geq 0 \Rightarrow a^2-b^2=(a+b)(a-b)$

$(k+4)(k-4) \geq 0$

$\Rightarrow k+4 \geq 0,\ k-4 \geq 0$

$\Rightarrow k \leq -4,\ k \geq 4$