PAIR OF LINEAR EQUATIONS

11 March 2026 05:13

Revision Notes on Pair of Linear Equations in two variables

Linear Equations

Powers → Variable - Maximum - 1

A Linear Equation is an equation of straight line. It is in the form of

$$ax + by + c = 0$$

$ax^2 + bx + c = 0$
$= 2$
quadratic

where a, b and c are the real numbers ($a \neq 0$ and $b \neq 0$) and x and y are the two variables.

Here a and b are the coefficients and c is the constant of the equation.

Pair of Linear Equations

Two Linear Equations having two same variables are known as the pair of Linear Equations in two variables.

$$a_1x + b_1y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

$\boxed{ax + by + c = 0}$ = degree = 1

$a \neq 0, b \neq 0, c$

variable variable constant - (number)

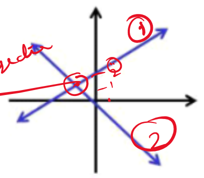
$a_1x + b_1y + c_1 = 0 \rightarrow (1)$
$a_2x + b_2y + c_2 = 0 \rightarrow (2)$ → 2 equation → Same

variable variable constant (number)

Graphical method of solution of a pair of Linear Equations

As we are showing two equations, there will be two lines on the graph.

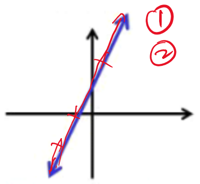
1. If the two lines **intersect** each other at one particular point then that point will be the only solution of that pair of Linear Equations. It is said to be **a consistent** pair of equations.



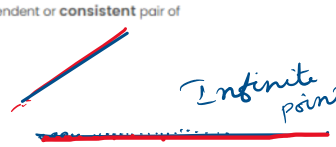
one solution

$(x, y) = (1, 2)$
only solution.

2. If the two lines **coincide** with each other, then there will be infinite solutions as all the points on the line will be the solution for the pair of Linear Equations. It is said to be dependent or **consistent** pair of equations.



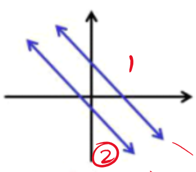
infinitely many solutions



Infinite points

3. If the two lines are **parallel** then there will be no solution as the lines are not intersecting at any point. It is said to be **an inconsistent** pair of equations.

Parallel
No solution
No intersection



no solution

Coincide ?
meet each other

y Interpretation of the pair of equations

Pair of Equations	Ratio Comparison	Graphical Representation	Algebraic Interpretation
$a_1x + b_1y + c_1 = 0$ (1) $a_2x + b_2y + c_2 = 0$ (2)	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Only one solution
$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinite solution Many solution
$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution

(1) - $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ = (1) → only one solution.

Many solution

$$① - \frac{a_1}{a_2} \neq \frac{b_1}{b_2} = ① \to \text{only one solution.}$$

$$② = \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = ② \text{ Coincident} \to \text{Many Solution} \to \text{Infinite solution}$$

$$③ = \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} = ③ \text{ Parallel line} \to \text{No solution}$$

Q. Solve the following pair of linear equations by the substitution method.

(i) $x + y = 14 ; x - y = 4$

Soln. $x + y = 14 \to ①$

$x - y = 4 \to ②$

$x = ②$, $y = ?$

$x - y = 4$

$\boxed{x = 4 + y}$ put in eqn ①

$x + y = 14$

$4 + y + y = 14$

$4 + 2y = 14$

$2y = 14 - 4$

$2y = 10$

$y = \frac{10}{2}$ $\therefore \boxed{y = 5}$ Put in eqn ①

$x + y = 14$

$x + 5 = 14$

$x = 14 - 5$

$\boxed{x = 9}$

$\therefore (x, y) = (9, 5)$

$x + y = 14$

$9 + 5 = 14$

LHS = RHS

(ii) $s - t = 3 ; \frac{s}{3} + \frac{t}{2} = 6$

Soln. $s - t = 3$ ... (i)

$\frac{s}{3} + \frac{t}{2} = 6 \to ②$

$\frac{s^{\times 2}}{3_{\times 2}} + \frac{t^{\times 3}}{2_{\times 3}} = 6$

LCM (3, 2) = 6

$\frac{2s + 3t}{6} = 6$

$2s + 3t = 6 \times 6$

$2s + 3t = 36 \to ③$

$s - t = 3 \to ①$

$2s + 3t = 36 \to ②$

$s - t = 3$

$\boxed{s = 3 + t} \to$ eqn ②

$2s + 3t = 36$

$2(3 + t) + 3t = 36$

$6 + 2t + 3t = 36$

$6 + 5t = 36$

$5t = 36 - 6$

$5t = 30$

$t = \frac{30}{5} =$

$\boxed{t = 6}$ Sub in eqn ①

$s - t = 3$

$s - 6 = 3$

$s = 3 + 6$

$\boxed{s = 9}$

$\therefore$ Solution $\boxed{t = 6, s = 9}$

1 Solve the following pairs of equations using the substitution method.
Check all solutions.

a $x + y = 3$ and $y = 4$
b $x + y = 7$ and $y = x + 3$
c $x + y = -3$ and $y = x + 1$
d $x - y = 5$ and $y = 1 - x$
e $2x + y = 9$ and $y = x - 3$
f $2x + y = 8$ and $y = x - 4$
g $2x - y = 10$ and $y = 10 - 3x$
h $x + 2y = 9$ and $y = 2x - 3$
i $2x + y = 14$ and $x = 6$
j $2x + y = 7$ and $x = y - 4$

1 Solve these equations:
a $x + (x + 4) = 6$ b $2x - (x + 3) = 5$
2 Substitute $y = x + 2$ for y in the equation $x + y = 10$.

a) $x + y = 3$, $y = 4$ ①

$x + (4) = 3$

$x = 3 - 4$

$\boxed{x = -1}$

b) $x + y = 7 \to ①$

$y = x + 3 \to ②$

$x + (x + 3) = 7$

$2x + 3 = 7$

$x = 7 - 3$

c) $x + y = -3 \to ①$

$y = x + 1 \to ②$

$x + (x + 1) = -3$

$2x = -3 - 1$

$x = \frac{-4}{2}$

d) $x - y = 5$

$y = 1 - x$

$x - (1 - x) = 5$

$x - 1 + x = 5$

$x = 3-4$
$\boxed{x=-1}$
$\boxed{y=4}$

$2x+3=7$
$x = \frac{7-3}{2}$
$= \frac{4}{2}$
$\boxed{x=2}$
$y = x+3$
$y = 2+3$
$\boxed{y=5}$

$2x = -3-1$
$x = \frac{-4}{2}$
$\boxed{x=-2}$
$y = x+1$
$y = -2+1$
$\boxed{y=-1}$

$x-(1-x)=5$
$x-1+x=5$
$2x = 5+1$
$x = \frac{6}{2}$
$\boxed{x=3}$
$y = 1-x$
$= 1-3$
$\boxed{y=-2}$

Question 1: Solve the following simultaneous equations by using elimination.

(a)	$6x + y = 18$ $4x + y = 14$	(b)	$4x + 2y = 10$ $x + 2y = 7$ $x=1, y=3$	(c)	$9x - 4y = 19$ $4x + 4y = 20$ $x=3, y=2$
(d)	$2x + y = 36$ $x - y = 9$ $x=15, y=6$	(e)	$6x - 3y = 12$ $4x - 3y = 2$ $x=5, y=6$	(f)	$3x - 6y = 6$ $2x - 6y = 3$ $x=3, y=\frac{1}{2}$
(g)	$8x + 7y = 39$ $8x + 2y = 34$ $x=4, y=1$	(h)	$x + 3y = 38$ $x + 6y = 53$ $x=23$ $y=5$	(i)	$6x + 3y = 48$ $6x + y = 26$ $x=5/2$ $y=11$
(j)	$2x - 4y = 10$ $2x + 3y = 24$ $x=9$ $y=2$	(k)	$5x - 2y = 120$ $5x + y = 165$ $x=30$ $y=15$	(l)	$x - 2y = 8$ $x - 3y = 3$ $x=18, y=5$
(m)	$3x + 2y = 54$ $2x - 2y = 16$ $x=14, y=6$	(n)	$7x - 4y = 80$ $3x - 4y = -80$ $x=40, y=50$	(o)	$5x - 2y = -23$ $5x - 6y = -39$ $x=-3$ $y=4$
(p)	$6x + 2y = -26$ $2x + 2y = -10$ $x=-4$ $y=-1$	(q)	$x - 5y = 65$ $2x - 5y = 85$ $x=20$ $y=-9$	(r)	$10x - 10y = -40$ $10x + 4y = 16$ $x=0$ $y=4$

$\cancel{6x} + 3y = 48$
$-\ \cancel{6x} + y = 26$

$2y = 22$
$\boxed{y=11}$ ✓

$6x + 3y = 48$
$6x + 3(11) = 48$
$6x = 48 - 33$
$6x = 15$
$x = \frac{15}{6} = \frac{5}{2}$
$\boxed{x = \frac{5}{2}}$ ✓

48
-33
15

a) $6x + y = 18 \quad \rightarrow (1)$
$4x + y = 14 \quad \rightarrow (2)$ Sub 2 from 1

$2x = 4$
$x = \frac{4}{2}$
$\boxed{x=2}$ Sub in eqn (1)

$6x + y = 18$
$6(2) + y = 18$
$12 + y = 18$
$y = 18 - 12$
$\boxed{y=6}$

Check
$4x + y$
$4(2) + 6$
$8 + 6 = 14$

r) $\cancel{10x} - 10y = -40$
$(-)\ \cancel{10x} + 4y = 16$

$-14y = -56$
$y = \frac{-56}{-14}$
$\boxed{y=4}$

$10x + 4y = 16$
$10x + 4(4) = 16$
$10x = 16 - 16$
$10x = 0$
$x = \frac{0}{10}$
$\boxed{x=0}$

Book sum

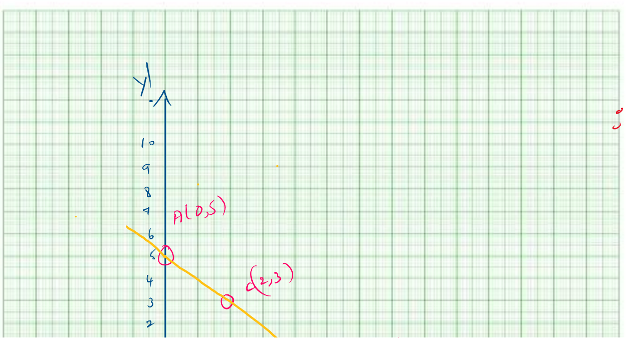
4) (i) $x + y = 5 \quad \rightarrow (1) \quad \Rightarrow a_1 = 1,\ b_1 = 1 \quad c_1 = 5$ $\qquad x + y - 5 = 0$
$2x + 2y = 10 \div 2$ $\qquad 2x + 2y - 10 = 0$
$x + y = 5 \quad \rightarrow (2) \quad = a_2 = 1 \quad b_2 = 1 \quad c_2 = 5$

$\frac{a_1}{a_2} = \frac{1}{1}, \quad \frac{b_1}{b_2} = \frac{1}{1} \quad \frac{c_1}{c_2} = \frac{5}{5} = \frac{1}{1}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow$

$\boxed{x + y = 5 \quad \text{or} \quad y = 5 - x}$

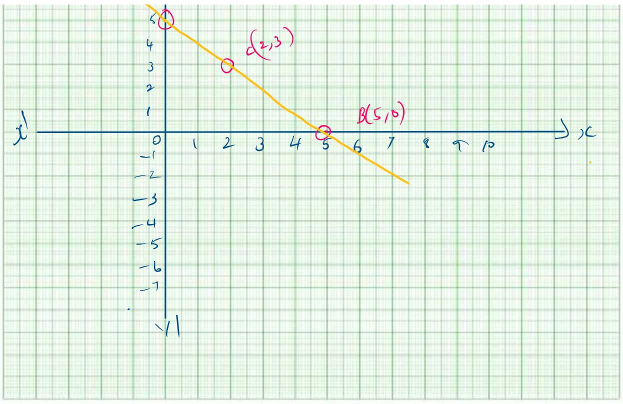
x	0	5	2
$y = 5 - x$	5	0	3
Points	A(0,5)	B(5,0)	C(2,3)



Here in Eqn (2) is same as eqn (1)

$\therefore$ table for both equation is same

$\therefore \Rightarrow$ These points on graph paper are coincident, so pair of linear equation have infinitely many solutions and hence consistent



equation have infinitely many solutions and hence consistent

(ii) $x - y = 8$, $3x - 3y = 16$

$x - y - 8 = 0 \to (1)$ $\quad 3x - 3y - 16 = 0 \to (2)$

On comparing eqn (1) & (2)

$a_1 = 1$, $b_1 = -1$, $c_1 = -8$

$a_2 = 3$, $b_2 = -3$, $c_2 = -16$

$\frac{a_1}{a_2} = \frac{1}{3}$, $\frac{b_1}{b_2} = \frac{+1}{+3}$, $\frac{c_1}{c_2} = \frac{\cancel{8}}{+\cancel{16}} = \frac{1}{2}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ = No solution

$\Rightarrow$ parallel line.

(iii) $3x - y = 3$; $9x - 3y = 9$ (Substitution)

$3x - y = 3 \to (1)$ $\quad \to$ $3x - y - 3 = 0 \to (1)$

$9x - 3y = 9 \to 2$ $\quad$ $9x - 3y - 9 = 0 \to (2)$

$a_1 = 3$, $b_1 = -1$, $c_1 = -3$

$a_2 = 9$, $b_2 = -3$, $c_2 = -9$

$\frac{a_1}{a_2} = \frac{\cancel{3}\,1}{\cancel{9}\,3}$, $\frac{b_1}{b_2} = \frac{+1}{+3}$, $\frac{c_1}{c_2} = \frac{+\cancel{3}\,1}{+\cancel{9}\,3} = \frac{1}{3}$

$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{3}$

* MCQ
* Assertion Reason
* CBQ

$3x - y = 3$

$3x = 3 + y$

$\boxed{x = \frac{3+y}{3}}$ $\to$ Sub in eqn (2)

$9x - 3y = 9$

$\cancel{9}^{3}\left(\frac{3+y}{\cancel{3}_1}\right) - 3y = 9 \Rightarrow 3(3+y) - 3y = 9$

$9 + \cancel{3y} - \cancel{3y} = 9$

$\boxed{9 = 9}$

This eqn has no variable but it is true for all values of x. So this pair of linear eqn has infinitely many solution.

(iv) HW $\quad 0.2x + 0.3y = 1.3 \quad \times 10 \quad = 2x + 3y = 13$

$0.4x + 0.5y = 2.3 \quad \times 10 \quad 4x + 5y = 23$

(v) $\sqrt{2}x + \sqrt{3}y = 0$; $\sqrt{3}x - \sqrt{8}y = 0$

$\sqrt{2}x + \sqrt{3}y = 0 \to (1)$

$\sqrt{3}x - \sqrt{8}y = 0 \to (2)$

Check

$(2) \to \sqrt{3}x - \sqrt{8}y = 0$

$\sqrt{3}(0) - \sqrt{8}(0) = 0$

$0 = 0$

$\sqrt{2}x + \sqrt{3}y = 0 \to (1)$

$\sqrt{2}x \quad -\sqrt{3}y$

$\sqrt{2}x+\sqrt{3}y=0 \rightarrow (1)$

$\sqrt{2}x=-\sqrt{3}y$

$\boxed{x=\frac{-\sqrt{3}y}{\sqrt{2}}}$ sub eqn (2)

(2)

$\sqrt{3}(0)-\sqrt{8}(0)=0$

$\sqrt{3}x-\sqrt{8}y=0$

$\sqrt{3}\left(\frac{-\sqrt{3}y}{\sqrt{2}}\right)-\sqrt{8}y=0$

$\sqrt{3}\times\sqrt{3}=3$

$\frac{-3y}{\sqrt{2}}-\sqrt{8}y=0 \Rightarrow \times\sqrt{2}$ on both sides

$\frac{\cancel{\sqrt{2}}(-3y)}{\cancel{\sqrt{2}}}-\sqrt{2}\sqrt{8}y=0\times\sqrt{2}$

$-3y-\sqrt{16}y=0$

$-3y-4y=0$

$-7y=0 \Rightarrow \boxed{y=0}$ in eq (1)

$y=\frac{0}{-7}$

$\sqrt{2}x+\sqrt{3}y=0$

$\sqrt{2}x+\sqrt{3}(0)=0$

$\sqrt{2}x=0$

$x=\frac{0}{\sqrt{2}}=0$

$\boxed{x=0}$

(vi) $\frac{3x}{2}-\frac{5y}{3}=-2$; $\frac{x}{3}+\frac{y}{2}=\frac{13}{6}$

$\frac{3x}{2}-\frac{5y}{3}=-2 \quad \times 6$

LCM (6)

$\cancel{6}^{3}\times\frac{3x}{\cancel{2}}-\cancel{6}^{2}\times\frac{5y}{\cancel{3}_1}=-2\times6$

$9x-10y=-12 \rightarrow (1)$

$\frac{x}{3}+\frac{y}{2}=\frac{13}{6} \quad \boxed{\times 6}$

$\frac{\cancel{6}^{2}x}{\cancel{3}}+\frac{\cancel{6}^{3}y}{\cancel{2}}=\frac{\cancel{6}\,13}{\cancel{6}}$

$2x+3y=13 \rightarrow (2)$

Take eqn (1)

$9x=-12+10y$

$\boxed{x=\frac{-12+10y}{9}}$ → Sub eqn 2

$2x+3y=13$

$2\left(\frac{-12+10y}{9}\right)+3y=13$

$\frac{-24+20y}{9}+3y=13 \quad \boxed{\times 9}$

$-24+20y+27y=117$

$-24+47y=117$

$47y=117+24$

$47y=141$

$y=\frac{141}{47}$

$\boxed{y=3}$

Sub in eqn (2)

$2x+3y=13$

$2x+3(3)=13$

$2x=13-9$

$2x=4$

$x=\frac{4}{2}=2$

$\boxed{x=2}$

$\therefore \boxed{x=2} \quad \boxed{y=3}$