

Example 2. Divide 16 into two parts such that twice the square of the greater part, exceeds the square of the smaller part by 164.
CBSE 2023 (Basic)

Let the larger and smaller parts be x and y respectively

According to the equation,

$$(x+y=16) \rightarrow 0$$

$$2x^2 = y^2 + 164 \rightarrow ②$$

From eqn ①, we get $x = 16 - y$ put in eqn ②

$$2(16-y)^2 = y^2 + 164$$

$$2(256 - 2(16)y + y^2) = y^2 + 164$$

$$\rightarrow 512 - 64y + 2y^2 = y^2 + 164$$

$$512 - 64y + 2y^2 - y^2 - 164 = 0$$

$$y^2 - 64y + 348 = 0$$

$$y^2 - 58y - 6y + 348 = 0$$

$$y(y-58) - 6(y-58) = 0$$

$$(y-58)(y-6) = 0$$

$$y = 58, y = 6$$

$$y = 6$$

$$\because y < 16$$

Put in eqn ①

$$x + y = 16$$

$$x = 16 - 6 = 10$$

$$x = 10$$

$\therefore$ The two natural numbers are 6, 10

Type V Word Problems Based on Geometry

Example 9. The hypotenuse of right angled triangle is 6 m more than twice the shortest side. If the third side is 2 m less than the hypotenuse, then find all sides of the triangle.
CBSE 2012, 11

Ans The length of the shortest side be 'x' m

$$\text{hypotenuse} = 2x + 6,$$

$$\text{If 3rd side is } = 2x + 6 - 2$$

2m less

$$\text{third side} = 2x + 4 \text{ m}$$

By Pythagoras theorem, $H^2 = P^2 + B^2$

hyp² = Sum of squares of other sides

$$(2x+6)^2 = x^2 + (2x+4)^2 \quad \because (a+b)^2 = a^2 + 2ab + b^2$$

$$4x^2 + 24x + 36 = x^2 + 4x^2 + 16x + 16$$

$$\rightarrow x^2 + 16x + 16 - 24x - 36 = 0$$

$$x^2 - 8x - 20 = 0$$

$$x^2 - 10x + 2x - 20 = 0$$

$$x(x-10) + 2(x-10) = 0$$

$$(x-10)(x+2) = 0$$

$$x-10=0, x+2=0$$

$$x=10, x=-2 \text{ length of the side cannot be negative}$$

$$\therefore x=10$$

Shortest side is 10 m

$$\text{hypotenuse} = 2x + 6 = 2(10) + 6 = 20 + 6 = 26 \text{ m}$$

$$\text{hyp} = 26 \text{ m}$$

$$\text{third side} = 2x + 4 = 2(10) + 4 = 24 \text{ m}$$

$$\text{third side} = 24 \text{ m}$$

Type IV Word Problems Based on Time and Work

Example 8. Two pipes together can fill a tank in $\frac{15}{8}$ hr.
The pipe with larger diameter takes 2 h less than the pipe with smaller diameter to fill the tank separately.
Find the time in which each pipe can fill the tank separately.
CBSE 2023 (Standard)

Let the time taken by smaller diameter is 'x' hr to fill the tank

→ Time taken by larger diameter = $(x - 2)$ h

In 1 hr, the tap with smaller diameter = $\frac{1}{x}$ part of the tank
1 hr, the tap with larger diameter = $\frac{1}{x-2}$ part of the tank

Totally tank fill = $\frac{15}{8}$ hrs.

Thus in 1 hr, the tap fill $\frac{8}{15}$ part of the tank $\frac{1}{\frac{15}{8}} = \frac{8}{15}$

$$\frac{1}{x} + \frac{1}{x-2} = \frac{8}{15}$$

$$\frac{x-2+x}{x(x-2)} = \frac{8}{15}$$

$$(2x-2)15 = 8(x^2-2x)$$

$$30x-30 = 8x^2-16x$$

$$8x^2-16x-30x+30=0$$

$$8x^2-46x+30=0 \quad \div 2$$

$$4x^2-23x+15=0$$

$$a=4, b=-23, c=15$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-(-23) \pm \sqrt{(-23)^2-4(4)(15)}}{2(4)}$$

$$= \frac{23 \pm \sqrt{529-240}}{8}$$

$$= \frac{23 \pm \sqrt{289}}{8}$$

$$= \frac{23+17}{8}$$

$$= \frac{23+17}{8}, \frac{23-17}{8}$$

$$x = \frac{40}{8}, \frac{6}{8}$$

$$x=5, \quad x=\frac{3}{4} \text{ (This is not applicable)} \Rightarrow x=5$$

∴ Smaller diameter tap will fill at 5 hrs

⇒ Larger diameter tap will fill at $x-2 = 5-2 = 3$ hrs

$$x=20, x=-20 \text{ (It is impossible)}$$

Ex: 4.3

③ L is 2B

Let breadth be x

then length is $l=2x$ m

According to the question,

$$\text{Area} = 800 \text{ m}^2$$

$$l \times b = 800 \text{ m}^2$$

$$2x(x) = 800$$

$$2x^2 = 800$$

$$x^2 = \frac{800}{2}$$

$$x^2 = 400$$

$$x = \sqrt{400}$$

$$x = 20$$

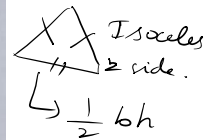
$$\frac{-5 \pm \sqrt{25 - 4}}{2}$$

Type VI Word Problems Based on Mensuration

Example 10. A rectangular park is to be designed whose breadth is 3 m less than its length. Its area is to be 4 sq m more than the area of a park that has already been made in the shape of an isosceles triangle with its base as the breadth of the rectangular park and of altitude 12 m. Find its length and breadth of the rectangular park.

CBSE 2016

$\square - l \times b$



$\text{Area} = \frac{\text{Base} \times \text{Height}}{2}$

Altitude = height

$b = l - 3$

Let breadth of $\square$ park be x m

$\checkmark b = x$
 $\checkmark l = x + 3$

$b = l - 3$
 $\hookrightarrow l = +3 + b$
 $l = x + 3$

Area of $\square$ park = $l \times b$
 $(x+3) \times x$
 $= x^2 + 3x \text{ m}^2$

Given Base of triangular park = breadth of the rectangular
 $B = x \text{ m}$

$h = 12 \text{ m}$
Area of triangular park = $\frac{1}{2} \times x \times 12 = 6x \text{ m}^2$

According to the question,

Area of rectangular = 4 + Area of triangular park

$x^2 + 3x = 4 + 6x$

$x^2 + 3x - 6x - 4 = 0$

$x^2 - 3x - 4 = 0$

$x^2 - 4x + 1x - 4 = 0$

$x(x-4) + 1(x-4) = 0$

$(x-4)(x+1) = 0$

$x-4=0$ $x+1=0$

$x=4$ $x=-1$ This is not possible
cannot be negative

$\therefore$ breadth = 4 m, $l = x + 3 \Rightarrow 4 + 3 = 7 \text{ m}$

Type VII Word Problems Based on Cost Price and Selling Price

Example 11. A dealer sells a toy for ₹ 24 and gains as much per cent as the cost price of the toy. Find the cost price of the toy.

Find CP = ? Let CP = x

SP = ₹ 24 $\Rightarrow$ $S.P = CP + \text{Gain}$

Profit (Gain) = $\frac{\text{Gain \%} \times CP}{100}$

original formula

$\text{Gain \%} = \frac{\text{Gain} \times 100}{CP}$

$\text{Loss \%} = \frac{\text{Loss} \times 100}{CP}$

Given gain % = C.P

$$\therefore \text{gain \%} = x$$

$$\text{Profit} = \frac{x \times x}{100} \Rightarrow \frac{x^2}{100}$$

$$\text{Now SP} = \text{CP} + \text{Gain}$$

$$24 = x + \frac{x^2}{100} \times 100$$

$$100(24) = 100x + x^2$$

$$2400 = 100x + x^2$$

$$x^2 + 100x - 2400 = 0$$

$$x^2 + 120x - 20x - 2400 = 0$$

$$\therefore \text{SP} = 24$$

$$\begin{array}{r} -2400 \\ \wedge \\ 120 - 20 \\ + 100 \end{array}$$

$$x(x+120) =$$

$$(x+120)(x-20)$$

$$x+120 = 0$$

$$x = -120$$

CP can not

$$\text{So } x = 20$$

Cost price

Exercise 4.4

- If two numbers differ by 2 and their product is 360, then find the numbers.
- If the product of two consecutive natural numbers is 210, then determine the numbers.
- The sum of two numbers is 15. If the sum of their reciprocals is $\frac{3}{10}$, find the two numbers.
CBSE 2023 (Standard)
- Two numbers differ by 3 and their product is 504. Find the numbers.
- The denominator of a fraction is 3 more than its numerator. The sum of the fraction and its reciprocal is $\frac{29}{10}$. Find the fraction.
- Divide 29 into two parts, so that the sum of the squares of the parts is 425.
- If the product of two consecutive odd numbers is 143, then find the numbers.
- Some students planned a picnic. The total budget for food was ₹ 2000. But 5 students failed to attend the picnic and thus the cost of food for each member increased by ₹ 20. How many students attended the picnic and how much did each student pay for the food?
- The perimeter of a right angled triangle is 70 units and its hypotenuse is 29 units. Find the length of the other sides.
- A person on tour has ₹ 360 for his expenses. If he extends his tour for 4 days, then he has to cut down his daily expenses by ₹ 3. Find the original duration of the tour.
- A rectangular field is 20 m long and 14 m wide. There is a path of equal width all around it, having an area of 111 sq m. Find the width of the path.
CBSE 2013, 12
- In a flight of 600 km, an aircraft was slowed down due to bad weather. Its average speed for the trip was reduced by 200 km/h and the time of flight increased by 30 min. Find the duration of the flight.
- The area of a right angled triangle is 480 cm². If the base of the triangle is 8 cm more than twice the height (altitude) of the triangle, then find the sides of the triangle.
CBSE 2013
- A two-digit number is such that the product of its digit is 35. When 18 is added to the number, the digits interchange their places. Find the number.
- 27 yr hence, Sanjay's age will be square of what it was 29 yr ago. Find his present age.

$$20(x+120)=0$$

$$-0)=0$$

$$)C-20=0$$

$$x=20$$

be negative

20

of the boy = $\neq 20$.

consecutive

$$\begin{array}{ccc} 1 & 2 & 3 & 4 \\ \downarrow & \downarrow & \downarrow & \\ x & x+1 & x+2 & \end{array}$$

$$2x+1 \quad 3x+1$$

$$1 \quad 3 \quad 5 \quad 7 \quad 9$$

$$\begin{array}{c} 1 \\ \downarrow \\ x \end{array} \boxed{x+2}$$

$$x(x+2) = 143$$

$$x^2 + 2x - 143 = 0$$

$$\underline{x^2 + 13x} \quad \underline{-17x - 143 = 0}$$

$$\boxed{-143}$$

$$\begin{array}{cc} 13 & -11 \\ +2 & \end{array}$$

$$x(x+13) + 11(x+13) = 0$$

$$(x+13)(x+11) = 0$$

$$\boxed{x = -13}, \quad x = 11 \quad \boxed{x = 11}$$

x can not be negative

The two numbers are x , $x+2$

11, 13