

1) The sum of the squares of three consecutive natural numbers is 110. Determine the numbers.

Let the three consecutive natural numbers be $x, x+1, x+2$

Squares of natural numbers $x^2, (x+1)^2, (x+2)^2$
 $\text{sum} = x^2 + (x+1)^2 + (x+2)^2 = 110$ (At 6) $\frac{1}{2} = a^2 + 2ab + b^2$

$$= x^2 + x^2 + 2x + 1 + x^2 + 4x + 4 = 110$$

$$= 3x^2 + 6x + 5 = 110$$

$$= 3x^2 + 6x + 5 - 110 = 0$$

$$\Rightarrow 3x^2 + 6x - 105 = 0$$

$$3(x^2 + 2x - 35) = 0 \quad \because 3 \neq 0$$

$$\Rightarrow \frac{1}{3}(x^2 + 2x - 35) = 0$$

This is required equation

Factorise:

$$-35$$

$$7-5$$

$$+5$$

$$x^2 + 7x - 5x - 35 = 0$$

$$x(x+7) - 5(x+7) = 0$$

$$(x+7)(x-5) = 0$$

$$x+7 = 0, x-5 = 0$$

$$x = -7, x = 5$$

But $x \neq -7$ because

x is natural number

$$\therefore x = 5$$

Hence the numbers are

$$x, x+1, x+2$$

$$5, 5+1, 5+2$$

$$5, 6, 7$$

Age type

2) 1 year ago, a man was 8 times as old as his son.

Now his age is equal to the square of his son's age. Find their present ages.

Ans

Let the present age of son be x yr

1 yr ago son's age = $(x-1)$ yr

1 yr ago man's age = $8(x-1)$
 $= (8x-8)$ yr

Present age of man = $8x-8+1$
 $= 8x-7$ yr

According to the question

$$8x-7 = x^2 \Rightarrow x^2 - 8x + 7 = 0$$

Factorise:

$$x^2 - 8x + 7 = 0$$

$$\begin{array}{c} 7 \\ -7 \end{array} \begin{array}{c} 1 \\ -1 \end{array}$$

$$x^2 - 7x - 1x + 7 = 0$$

$$x(x-7) - 1(x-7) = 0$$

$$(x-7)(x-1) = 0$$

$$x-7 = 0, x-1 = 0$$

$$x = 7, x = 1$$

But $x=1$ is not possible
 $\therefore x=7$

Present age of son

$$= 7 \text{ yr}$$

Present age of father

$$8x-7$$

$$8(7)-7$$

$$56-7$$

$$= 49 \text{ yr}$$

$\therefore$ Son's age = 7 yr

Father's age = 49 yr

3) Time, distance, speed $\times \times \times$.

Type III Word Problems Based on Time, Distance and Speed

Example 6. A train travels at a certain average speed for a distance of 54 km and then travels a distance of 63 km at an average speed of 6 km/h more than the first speed. If it takes 3h to complete the journey, what was its first average speed?

CBSE 2023 (Standard)

Ans

Let original average speed be x km/hr

Time taken by train to cover 54 km $\left\{ = \frac{D}{S} = \frac{54}{x} \right.$

Time taken by train to cover 63 km with increased speed $\left\{ = \frac{63}{x+6} \right.$

6 km more than original speed

$$\underline{\underline{x+6}}$$

According to the question

$$\frac{54}{x} + \frac{63}{x+6} = 3$$

$$\frac{54(x+6) + (63)x}{x(x+6)} = 3$$

$$54(x+6) + 63x = 3x(x+6) \quad \div 3$$

$$x^2 - 33x - 108 = 0$$

$$a = 1, b = -33, c = -108$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-33) \pm \sqrt{(-33)^2 - 4(1)(-108)}}{2(1)}$$

$$\begin{array}{r} 36-3 \\ \hline 33 \\ \hline 1089 \\ 432 \\ \hline 1521 \end{array}$$

$$\begin{array}{r} 2 \overline{) 108} \\ 2 \overline{) 54} \\ 3 \overline{) 27} \\ 3 \overline{) 9} \\ \hline 3 \end{array}$$

$$54(x+6) + 63x = 3x(x+6) \quad \div 3$$

$$18(x+6) + 21x = x(x+6)$$

$$18x + 108 + 21x = x^2 + 6x$$

$$x^2 + 6x - 39x - 108 = 0$$

$$\Rightarrow x^2 - 33x - 108 = 0$$

$$x^2 - 36x + 3x - 108 = 0$$

$$x(x-36) + 3(x-36) = 0$$

$$(x-36)(x+3) = 0$$

$$\boxed{x=36}, \boxed{x=-3}$$

$$2(1)$$

$$= \frac{33 \pm \sqrt{1089 + 432}}{2}$$

$$= \frac{33 \pm \sqrt{1521}}{2}$$

$$= \frac{33 \pm 39}{2} \Rightarrow \frac{33+39}{2}, \frac{33-39}{2}$$

$$x=36 \quad x=-3$$

Speed can't be negative
so $x=36$

Example 7. A motor boat whose speed is 18 km/h in still water takes 1 h more to go 24 km upstream, than to return to the same point. Find the speed of the stream and total time of the journey. CBSE 2023 (Basic)

Sol. Given, the speed of the boat in still water = 18 km/h. ✓

Let the speed of the stream be x km/h.

Speed of the boat upstream

= Speed of boat in still water - Speed of the stream

∴ Speed of the boat upstream = $(18 - x)$ km/h

Speed of the boat downstream = Speed of boat in still water

+ Speed of the stream

∴ Speed of the boat downstream = $(18 + x)$ km/h

Time of upstream journey

= Time for downstream journey + 1 h

∴ $\frac{\text{Distance covered upstream}}{\text{Speed of the boat upstream}}$

= $\frac{\text{Distance covered downstream}}{\text{Speed of the boat downstream}} + 1 \text{ h}$

$(x-y)$
 $(x+y)$

$$\frac{24}{18-x}$$

$$\frac{24}{18-x} - \frac{24}{18+x}$$

$$\frac{24(18+x) - 24(18-x)}{(18-x)(18+x)}$$

$$24(18+x) - 24(18-x)$$

$$432 + 24x$$

$$480$$

$$x^2 + 48x$$

$$x^2 + 5x$$

$$x(x + 5)$$

$$\begin{array}{r} 54 \overline{) 324} \\ 54 \overline{) 324} \\ \hline 48 \end{array}$$

$$= \frac{24}{18+x} + 1$$

$$= 1$$

$$f(18-x) = 1$$

$$\frac{24}{18-x} = \frac{(a-b)(a+b)}{a^2-b^2}$$

$$24(18-x) = (18-x)(18+x)$$

$$x - 432 + 24x = (18)^2 - (x)^2$$

$$x = 324 - x^2$$

$$8x - 324 = 0$$

$$54x - 6x - 324 = 0$$

$$+54) - 6(x+54) = 0$$

Time of

$$\frac{24}{18-x} =$$

Time of d

24

2

upstream

$$\frac{24}{18-6} = \frac{24}{12}$$
$$= 2 \text{ hrs}$$

downstream

$$4 - 2x \dots$$

$$\begin{array}{r}
 2 \overline{) 324} \\
 \underline{162} \\
 81 \\
 \underline{63} \\
 18 \\
 \underline{18} \\
 0
 \end{array}$$

$$x(x$$

$$(x+$$

$$x+$$

$$x$$

Thus

$$(x+54) - 6(x+54) = 0$$

$$54)(x-6) = 0$$

$$54 = 0, \quad x-6 = 0$$

$$x = -54, \quad \boxed{x=6}$$

Speed can't
be negative

$$\text{Speed of stream} = \underline{\underline{6 \text{ km/h}}}$$

$$\begin{array}{r} 24 \\ \hline 18+x \end{array} = \frac{24}{18}$$

Total Time
up + d
2+1

∴ Total time
is 3 h

$$\frac{q}{3+6} = \frac{2q}{2q} = 1h$$

me
run

= 3 hrs

of journey
is